ECON 239 Term Paper: Limits to Arbitrage Literature Review

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1 Introduction

The field of behavioral finance is underpinned by the empirically proven fact that human agents do not adhere to theoretically optimal behavior. Literature on overconfidence, bubbles, herding effects, and much more would not be relevant if people acted in line with perfectly rational, risk-neutral agents. To quote Isaac Newton: "I can calculate the motion of heavenly bodies, but not the madness of people". Indeed, this so-called madness of people has led to thousands of papers trying to understand everyday financial happenings, and stock market aberrations is one of the most lucrative of these happenings.

The Efficient Market Hypothesis (EMH), first introduced in 1900 by Louis Bachelier [2] and popularized with multiple forms in 1970 by Eugene Fama [10], states that the price of assets represent all (or some, as proposed by Fama) of the publicly available information about them. The idea behind the hypothesis is that new information, such as an earnings report or a merger, should directly influence the price of the asset near instantaneously, causing the price to be fully informed. Portions of the EMH were tested and verified by Fama in his 1965 [9] and 1969 [11] papers, and the theory prevailed for several years, leading to the development of pricing models based on the EMH such as the Capital Asset Pricing Model and Fama-French Factor Model.

Despite the EMH's popularity, later literature found evidence against its claims. Grossman and Stiglitz (1980) [13] argued the impossibility of having a market that consisted only of fully informed traders; Shiller (1981) [20] found empirical evidence of excess volatility in stock prices; DeBondt and Thaler (1985) [8] discovered that investors overreact to news events. With mounting evidence against the EMH, Fischer Black's 1986 paper *Noise* [4] shined further light on the issues of the EMH and posited the existence of noise traders who were uninformed, irrational, and driving the unexpected behavior of asset prices. Because of these noise traders, prices could not be fully informed despite the opportunity to arbitrage away differences between the current price of an asset and its true value. Therefore, there must exist some limit to how much arbitrage is possible, but what mechanisms limit arbitrage, and how much is arbitrage is limited? From these questions, a new subfield of financial economics was born: arbitrage limits.

In the decades that followed, three main types of limits to arbitrage emerged. The first type of limit is cost, either due to market frictions, trading implementations, or other related factors. A famous paper discussing cost as a limit to arbitrage is Jones and Lamont (2002) [15] in which they analyze how stock prices are affected by the shorting costs. Simply put, if a stock is overpriced and arbitrageurs face high costs to short the stock, then the stock will remain overpriced until the cost lowers to a manageable level. Jones and Lamont use subsequent returns as a measure of how overpriced a stock is. Low returns on a stock indicate that the stock is overpriced. Using data from a "loan crowd" which shorted stocks amongst themselves from 1926 to 1933, the authors find that stocks that were recently added to the list of shortable stocks had lower subsequent returns due to their entrance at a high shorting cost. Thus, this shorting cost hampered arbitrageurs' ability to do away with mispricings.

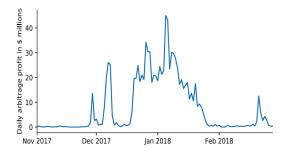
The second type of limit is risk. One component of this risk was that asset prices exhibited excess volatility, rendering trading strategies riskier than they would have been without the existence of noise traders. Da, Nagel, and Xiu (2024) [7] provide a modern, computational take on this aspect of risk. Using Arbitrage Pricing Theory, they find that many portfolios that have positive alphas cannot be realized due to the low statistical significance of their alphas caused by excess volatility, meaning that these arbitrage opportunities are invisible to arbitrageurs. The other component of risk that limits is the risk aversion of traders. Agents are often risk averse and are thus unwilling to pursue strategies that have a higher expected value if the variance of the strategy is also high. Markowitz (1952) [18] popularized a parameterization for risk aversion known as the mean-variance rule, and literature on the limits to arbitrage often cites risk aversion as a large reason why arbitrage opportunities can persist, using the mean-variance rule in their analysis.

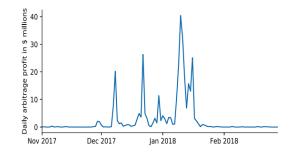
The third type of limit is agency, specifically in performance-based arbitrage. The most well-known paper in this literature is Shleifer and Vishny (1997) [21]. Shleifer and Vishny propose a three-period model of noise and sophisticated traders. In this model, the price is randomly distributed for the first two periods before it goes to its fundamental value in the third period. Sophisticated traders, who get their capital from outside sources, face a dilemma of how much to buy or sell if the asset is underpriced or overpriced in the first period, respectively. If they over-arbitrage, then the mispricing might worsen, signaling a worse performance to their investors and leaving them with less capital to correct the mispricing. Arbitrageurs therefore cannot fully correct mispricings due to the risk of losing capital, creating an imposed limit of arbitrage.

In the last decade, the literature on limits to arbitrage has slowed down as research on behavioral finance has moved to other fields, such as cognitive biases and household finance. However, there is still active research being done on arbitrage limits. This literature review focuses on two recent papers – Hautsch et al. (2024) [14] and Tian and Wu (2023) [23] – to see where further limits to arbitrage can be found.

2 Limits to Arbitrage in Cryptocurrency

Cryptocurrency is a modern form of fiat currency that has the primary benefit of being decentralized; that is, cryptocurrencies are not issued by any governing body and, in most cases, have a set amount of supply that cannot be changed. Cryptocurrency is stored on private wallets which store passwords and keys that enable users to access their cryptocurrency. To pay with a cryptocurrency, users transfer the cryptocurrency from one wallet to another. Each transfer is known as a transaction. Once a transaction is initiated, it is placed in a queue to wait to be verified. The queue is known as the mempool. Transactions are placed onto the mempool in blocks so that multiple transactions can be verified at the same time. The verification process requires decentralized computers to solve a cryptographic problem, and these decentralized computers, also known as miners, receive a small reward for verifying the transaction. After the transaction is verified, the cryptocurrency is moved





Panel A: US and Korea

Panel B: US and Japan

Figure 1: Price differences in bitcoin between international exchanges. Sourced from Makarov and Schoar (2020) [17].

into the recipient's wallet.

Financial markets surrounding cryptocurrency have existed ever since the advent of cryptocurrency and have only skyrocketed in popularity since then. Despite the activity surrounding cryptocurrency, mispricings persist. Cryptocurrency is traded on exchanges that handle matching buyers and sellers together to perform transactions. Exchanges often have pricing discrepancies that break the law of one price – the bid price on one exchange might be lower than the ask price on another for months – so there must exist market frictions that allow these opportunities to persist.

Makarov and Schoar (2020) [17] was one of the first papers to provide evidence of these mispricings and to suggest reasons why they exist. They used data from Kaiko, a private firm that collected cryptocurrency data since 2014, and analyzed violations of the law of one price. They found that mispricings persisted for months, citing three technical constraints as a large reason why arbitrageurs could not take advantage of arbitrage opportunities.

The first constraint was the existence of transaction fees. Each transaction requires the payment of a fixed fee to the miners who verifies it along with a percentage fee to the exchange on which the transaction was performed on. This percentage fee makes large-scale arbitrage too costly and dissuades arbitrage. The second constraint was governance risk. When traders trade on exchanges, they must transfer cryptocurrency from their wallet to a wallet on the exchange. If the exchange gets hacked or otherwise fails, then that cryptocurrency is lost, and the risk of this loss is what is known as governance risk. This risk effectively acts as another cost that arbitrageurs must pay and limits the possibility of arbitrage. The third constraint was cross-border capital controls. The exchanges with the largest observed price differences were ones that operated from closed countries. Closed countries are ones that disallow large transfers of currency out of the country. For example, Korea has laws mandating requests to move sums of money larger than \$50,000 USD out of the country which can be rejected. The fact that profits are hard to realize further limits arbitrage opportunities.

Makarov and Schoar (2020) also explain some intuitive arbitrage strategies that would not work as a

result of these constraints. One strategy involves short selling cryptocurrency on the more expensive exchange (denoted as the high exchange) and buying on the cheaper exchange (denoted as the low exchange). This strategy would not work because a majority of exchanges do not allow for short sales, and the ones that do are low exchanges. Another suggested strategy is to trade on margin to sell on the high exchange and buy on the low exchange. This strategy opens the arbitrageur up to convergence risk which significantly hampers arbitrage, as documented by Shleifer and Vishny (1997) [21]. The last discussed strategy is to hold bitcoins on both the low and high exchanges. The arbitrageur would sell on the high exchange and buy on the low exchange once a price difference is observed, transferring the bitcoins from the low exchange to the high exchange afterwards. This strategy faces issues with the time it takes to transfer bitcoins. During the transfer time, further mispricings may arise, so arbitrageurs are incentivized to wait for the maximum mispricing before executing their arbitrage strategy. This causes a bubble effect as arbitrageurs wait for the point right before the mispricing disappears, as documented in Abreu and Brunnermeier (2003) [1].

Hautsch, Scheuch, and Voigt (2024) [14] build upon Makarov and Schoar (2020), creating a theoretical framework for limits to cryptocurrency arbitrage (specifically for bitcoin) and empirically verifying it. The main two issues they study are settlement latency and trust in exchanges. Settlement latency refers to the time it takes for a bitcoin transaction to be verified and completed. They propose two methods of arbitrage in bitcoin markets: cross-exchange arbitrage and inventory arbitrage.

We begin with defining the profit that can be made from a violation of the law of one price. The instantaneous profit δ_t that can be made at any time t is given by

$$\delta_t := \max\{b_t^i - a_t^j, b_t^j - a_t^t, 0\} > 0 \tag{1}$$

where b_t^k and a_t^k represent the bid and ask prices at time t for a cryptocurrency on exchange k.

Cross-exchange arbitrage involves purchasing bitcoins on the low exchange, transferring it to the high exchange, and selling it there. This strategy faces the issue of settlement latency, as the arbitrageur must wait for the bitcoin transfers to be verified before being able to fully execute the buy and sell orders. Let τ be the known settlement latency time. Assuming that the log price change of bitcoin follows Brownian motion without drift, the log return of the arbitrage transaction from time t to time $t + \tau$ is given by

$$r_{(t:t+\tau)} := b_{t+\tau}^s - a_t^b = b_t^s - a_t^b + b_{t+\tau}^s - b_t^s = \delta_t + \sqrt{\tau}z$$
 (2)

where $z \sim N(0, \sigma^2)$. Note that this is the same as equation (1) for instantaneous profit but with an additional noise term.

This noise can cause risk averse traders to be dissuaded from pursuing the strategy. Using mean-variance preference from Markowitz (1952) [18], the arbitrageur's utility function is given by

$$CE = \mathbb{E}(r_{(t:t+\tau)}) - \frac{\gamma}{2} \mathbb{V}(r_{(t:t+\tau)}) = \delta_t - \frac{\gamma}{2} \sigma^2 \tau \ge 0 \Leftrightarrow \delta_t \ge \frac{\gamma}{2} \sigma^2 \tau.$$
 (3)

We see that the trader is more willing to pursue cross-exchange arbitrage when the instantaneous profit

is high, their risk aversion is low, the variance of bitcoin prices is low, or the settlement latency time is low.

Inventory arbitrage is the strategy described before of holding bitcoins on two separate exchanges, buying on the high exchange, and selling on the low exchange when a price difference is observed. This strategy faces default risk. Since the trader must transfer their bitcoins to both exchanges in order to perform this strategy, they run the risk of having their bitcoins stolen while they wait for an observable price difference: this risk is the default risk. Default risk means that profits will decrease proportional to wealth, as a share of the arbitrageur's bitcoins held on exchanges can be stolen at any time. Using Blais et al. (2019) [3] as a baseline, the fraction of deposits that are stolen c can be accounted for by defining the expected profit from inventory arbitrage as $\delta_t - \mathbb{E}(c)$ with known variance $\mathbb{V}(c) = \sigma_c^2$.

Assuming that bitcoin price dynamics are independent of default risk such that Cov(z, c) = 0, the arbitrageur wants to maximize their profits by allocating money into cross-exchange arbitrage (x_{τ}) , inventory arbitrage (x_c) , and a risk-free asset with returns r_f . Their optimization problem is thusly

$$\max_{x_{\tau}, x_c} = (x_{\tau} + x_c)(\delta_t - r_f) - x_c \mathbb{E}(c) - \frac{\gamma}{2} (x_{\tau}^2 \sigma^2 \tau + x_c^2 \sigma_c^2).$$
 (4)

Solving this problem gives the solutions of

$$x_{\tau} = \frac{\delta_t - r_f}{\gamma \sigma^2 \tau} \text{ and } x_c = \frac{\delta_t - \mathbb{E}(c) - r_f}{\gamma \sigma_c^2},$$
 (5)

meaning that the optimal amount not invested in the risk-free asset is

$$x_{\tau} + x_{c} = \frac{1}{\gamma \sigma_{c}^{2}} \left(\frac{\sigma_{c}^{2} + \sigma^{2} \tau}{\sigma^{2} \tau} (\delta_{t} - r_{f}) - \mathbb{E}(c) \right). \tag{6}$$

We see that arbitrageurs will invest more into bitcoin arbitrage when they have a low risk aversion, when the instantaneous price difference is high, when the risk-free return rate is low, when the settlement latency is low, when the volatility of bitcoin prices and default risk are low, and when the expected value of default risk is low. However, this static analysis reveals a core issue with arbitrage in bitcoin markets: there is no way to reduce default risk without increasing settlement latency. As stated before, transactions are verified by being placed into blocks, and the more blocks that are used to verify a transaction, the more secure the transaction becomes.

Empirical analysis reveals the exchanges with the lowest arbitrage bounds are the ones that require the least amount of blocks to verify a transaction, making them less secure. The inverse is also true; the exchanges with the highest arbitrage bounds are the most secure. Thus, efficient cross-exchange arbitrage and inventory arbitrage cannot exist together unless trust in bitcoin exchanges is increased without an increase in settlement latency.

3 Limits to Arbitrage in Options Writing

An option is a contract to buy or sell an asset at a later date for an agreed upon price. Options are also known as derivatives because they derive their value from the fundamental asset. Options pricing is dominated by the Black-Scholes (BS) model, formulated in Fischer Black et al. (1973) [5].

This proof is taken from UCI Professor Donald Saari's textbook Mathematics of Finance: An Intuitive Introduction [19]. The BS model draws from Bachelier (1900) [2] and begins by defining the price of an asset S as following a random walk with trend and volatility components:

$$dS = \mu S dt + \sigma S dW \tag{7}$$

where $dW \sim N(0, dt)$. This formula can be used to eliminate the risk from an option. First, construct a portfolio Π that is long an option and short some amount of the underlying asset:

$$\Pi(S,t) = V(S,t) - \Delta S. \tag{8}$$

Next, we will define the Ito's Lemma formula for a Taylor series expansion of a function of two variables as

$$df(S,t) = \sigma S \frac{\partial f}{\partial S} dW + \left[\mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \frac{\partial f}{\partial t} \right] dt.$$

(9)

Applying equation (3) to equation (8) gives us

$$d_{market}\Pi(S,t) = \left(\frac{\partial V}{\partial S} - \Delta\right)\sigma S dW + \left[\left(\frac{\partial V}{\partial S} - \delta\right)\mu S + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t}\right] dt. \tag{10}$$

If we set $\Delta = \frac{\partial V}{\partial S}$, we remove the random walk component from our formula. This strategy is known as delta-hedging: it removes underlying risk from the asset, resulting in

$$d_{market}\Pi(S,t) = \left[\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t}\right] dt. \tag{11}$$

We assume that the market has no arbitrage opportunities. This means that equation (11), which represents the possible returns from the options market, must be equal to the risk-free rate r from safe investments (like banks). With this, we derive the BS formula for the price of an option V as follows:

$$\begin{split} \big[\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2} + \frac{\partial V}{\partial t}\big]dt &= r\Pi dt \\ \big[\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2} + \frac{\partial V}{\partial t}\big]dt &= r\left(V - \frac{\partial V}{\partial S}S\right)dt \\ \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2} + \frac{\partial V}{\partial S}S - rV &= 0 \end{split}$$

We then solve this formula using the heat equation. For the purposes of the following analysis, we define the BS formula for a European option as

$$B(t, S_t, I_t) = \Delta_t S_t - S_t N(d_t) + K N(d_t - \frac{1}{2} I_t \tau)$$
(12)

where Δ_t is the amount of stock delta-hedged, I_t is the implied volatility, $N(\cdot)$ is the Normal cdf, $\tau = T - t$, and

$$d_t = \frac{\ln(S_t/K) + \frac{1}{2}I_t^2\tau}{I_t\sqrt{\tau}}.$$

Tian and Wu (2023) [23] analyze the BS options pricing model bottom-up, looking at the original formula and determining sources of risk that could deter arbitrageurs and generate mispricings. They utilize the Taylor series expansion of equation (12) to find these other sources of risk.

Define the excess return of an option position as

$$r_{t,T} = \frac{B(T, S_T, I_T) - B(t, S_t, I_t)}{S_t}.$$
(13)

Rewriting equation (13) as a Taylor expansion gives

$$r_{t,T} = \frac{B_t}{S} \Delta t + B_S \frac{\Delta S}{S} + \frac{1}{2} S B_{SS} \left(\frac{\Delta S}{S}\right)^2 + \frac{1}{3!} S^2 B_{SSS} \left(\frac{\Delta S}{S}\right)^3 + \frac{1}{4!} S^3 B_{SSSS} \left(\frac{\Delta S}{S}\right)^4 + \frac{B_I}{S} \Delta I + \cdots$$

$$(14)$$

where $B_t, B_S, B_{SS}, B_{SSS}, B_{SSSS}, B_I$ represent the partial derivatives of B with respect to t, S_t, I_t . These partial derivatives also represent different risks that traders are exposed to when they write an option.

Delta hedging is often not feasible because of technical constraints such as costs to perform a trade. Because option writers cannot perfectly delta hedge, they are still exposed to some of the underlying risk that comes from dW. This exposure is captured in the gamma term B_{SS} . Instead of delta hedging at every single moment, a common strategy is to reblanace the delta hedge once a sufficient volume of stock has been traded. In accordance with this strategy, Tian and Wu define the delta hedging cost HC as

$$HC_{t,i} = \sigma_{t,i} \sqrt{(1 - \rho_{t,i}^2)/DV_{t,i}}$$
 (15)

where $\sigma_{t,i}$ denotes the stock's historical return volatility estimator, $\rho_{t,i}$ denotes the return correlation of the stock with the market portfolio, and $DV_{t,i}$ denotes the average dollar trading volume on the stock.

Though delta hedging can remove risk in the underlying security, a delta-hedged option is still exposed to movements in stochastic volatility. That is, the fact that the return volatility of the underlying security is always changing creates inherent risk in options writing, especially since the BS model prices options based on the implied volatility at the time. This is known as volatility risk and is captured in the vega term B_I . Tian and Wu approximate volatility risk (VR) using a historical volatility estimator based on changes in the implied volatility in the past month:

$$VR_{t,i} = \omega_{t,i} \tag{16}$$

Delta hedging gets rid of small aberrations in stock prices, but it cannot deal with large jumps due to the BS model's use of the Normal distribution which has thin tails. These large jumps mainly affect deeply out-of-the-money options, as shown by Buraschi and Jackwerth (2001) [6]. The prevalence of these jumps can be estimated using excess kurtosis; Tian and Wu's measure of jump risk (JR) is given by

$$JR_{t,i} = \kappa_{t,i}\sigma_{t,i} \tag{17}$$

which is estimated using daily stock returns over the past month.

Options and their underlying securities typically have greater returns than the risk-free rate due to risk premiums. Tian and Wu control for these risk premiums by adding them into the regression. Specifically, they account for the historical risk premium, which is calculated by averaging the average returns of options over the past year, and the volatility risk premium, which is calculated by finding the difference between the implied volatility of an option and its predicted volatility.

After defining these risks and costs, Tian and Wu run a regression analysis to see their effects on options pricing. They find that delta hedging cost, volatility risk, and jump risk all significantly contribute to both the volatility of options and to the risk premiums associated with options. Because these risks are essentially unavoidable, they pose limits to the amount of arbitrage that options arbitrageurs can perform, leading to consistent mispricings.

4 Extensions: Arbitrage in Virtual Item Markets

Literature on virtual markets has been scarce. There are some papers on theoretical frameworks behind pricing strategies for game developers [12] [26] and empirical studies on what causes games to have valuable items [25], but there are almost no papers on how players price and sell items among each other.

The most well-known paper that studies players' pricing strategies is Ke et al. (2012) [16]. This paper analyzes the pricing strategies of creators in Second Life, a popular video game where players can build their own virtual worlds. Players can create their own virtual items in the game (e.g. furniture and homes) and put them for sale on the marketplace. Marketplace listings come with a caveat; purchased items may or may not have the full suite of permissions attributed to them. For example, players can sell a vase of flowers object and add the modify permission so that buyers can change the color of petals, or they could sell a chair object and add the copy permission so buyers can place multiple of them within their houses. Ke et al. use a fixed effects linear regression to determine whether or not the permissions granted to an object affected the price of the listing. They found that the permissions granted did have a statistically significant effect on the price of a listing across different listing types (e.g. animals, apparel, vehicles).

Using Ke et al. (2012) as a starting point, Vo (2025) [24] constructed a fixed effects model to look at the

effects of various factors on virtual items from the popular game *Counter-Strike 2*. Factors included in the model were item rarity, weapon type, release date, various attributes, and, most important for this discussion, the market on which it was sold on. After controlling for all other factors, the regression analysis found that the market on which an item was sold was highly statistically significant in determining its price. However, this observation directly violates the law of one price: prices should be the same on all markets, so the market on which an item is sold should not affect its price.

The market frictions in this virtual item market are very similar to the market frictions in the cryptocurrency market. When trying to attempt the equivalent of cross-exchange arbitrage (i.e. purchase an item on a low market and sell it on the high market), traders face a large amount of settlement latency. For Counter-Strike 2, a transaction proceeds as follows. First, the player purchases the item on the low market and waits for a script bot to give them the item they've purchased. This wait for the script bot can take anywhere from 15 minutes to an hour. Then, they must sell the item on the high market and wait for yet another script bot to retrieve the item they've sold. These considerations make the settlement latency for these transactions significant, and it worsens if the markets only accept payments in cryptocurrency which has become increasingly common.

Inventory arbitrage is also very difficult to perform. Some items are exceptionally rare, and only a few copies of them are available in the game. These items can also be some of the most mispriced ones, so the inability to perform inventory arbitrage hampers the accuracy of their pricing. Additionally, default risk comes into play with markets. Because most markets are third-party and not directly run by the game developers, players might not be willing to do trade with them unless they know for sure their transaction will go through. However, settlement latency does not need to increase in order to increase trust in a virtual item market. In fact, most trusted markets tout having a fast transaction time. Therefore, this market could theoretically be very efficient if the technology for transactions improved and reduced the time it took to perform transactions.

This theoretical efficiency is quickly dismissed by one fact: transaction costs are too high to perform arbitrage. The developer-backed market for items takes a 15% cut of all sales, and third-party markets also take hefty cuts in order to stay in business. Until this large market friction is removed, arbitrage in Counter-Strike 2 virtual item markets will be mostly ineffective. However, this analysis is completely theoretical; an interesting research idea would be to attempt to apply the arbitrage strategies discussed above to see whether profit could be made or not.

As financial markets continue to evolve, it is important to remember the limits to how informed markets can be. Many analysts believe that, despite all of the technological advancements made during the dot-com bubble to enhance the quality and quantity of information, the EMH partly caused the 2008 financial crisis. As one trader put it, "you had to be a fanatic to believe in the literal truth of the EMH" [22]. Humans are inevitably flawed creatures; no one will ever be perfectly rational and risk neutral, much to the detriment of people like Isaac Newton. In a way, these limits to arbitrage help teach us the greatest life lesson we can take away from behavioral finance: nobody is perfect.

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